Oak Ridge High School
SUMMER REVIEW PACKET

For students entering AP Calculus AB

Name: ____ Due Date: _______________

1. This packet is to be handed in to your Calculus teacher on the first day of the school year.

2. All work must be shown in the packet OR on separate paper attached to the packet.

3. Completion of this packet is worth 35 points entered as a quiz grade.

4. Unless specifically noted, work should be done *without* using your TI.

5. Please underline final answers with a highlighter.

FYI: An additional source of review of Algebra concepts can be found at the Khan Academy website.

Summer Review Packet for Students Entering Calculus

A. Complex Fractions

When simplifying complex fractions: multiply by a term that is equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators (baby fractions) in the complex fraction.

Examples:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

B. Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $\left(f \circ g\right)(x) = f(g(x))$ *or* $f\left(g(x)\right)$ read "***f*** of ***g*** of ***x***" means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find the following.**

6. $f(2) =$

7. $g(-3) =$

8. $f(t+1) =$

9. $f\left[g(-2)\right] =$

10. $g\left[f(m+2)\right] =$

11. $f\left(g\left(\sqrt{x}\right)\right) =$

12. $f(x+h) - f(x) =$

13. $g\left(x+h\right) - g\left(x\right) =$

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$**. Find the following.**

14. $h\left[f(-2)\right] =$

15. $f\left[g(x-1)\right] =$

16. $g\left[h(x^3)\right] =$

Find $\frac{f(x+h)-f(x)}{h}$, the Difference Quotient, for the given function *f*.

17. $f(x) = 9x + 3$

18. $f(x) = 5 - 2x^2$

C. Intercepts and Points of Intersection

To find the *x*-intercepts, let $y = 0$ in your equation and solve.
To find the *y*-intercepts, let $x = 0$ in your equation and solve.
Write the intercept as an ordered pair.

Example: $y = x^2 - 2x - 3$

$x - \text{int.}\ (Let\ y = 0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x - \text{intercepts}\ (-1,0)\ and\ (3,0)$

$y - \text{int.}\ (Let\ x = 0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept}\ (0,-3)$

Find the *x* and *y* intercepts for each.

19. $y = 2x - 5$

20. $y = x^2 + x - 2$

21. $y = x\sqrt{16 - x^2}$

22. $y^2 = x^3 - 4x$

D. Systems

Use the substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y^2 - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Add the two equations.

<u>Elimination Method</u>

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ and $x = 5$

Plug $x = 3$ and $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\qquad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\qquad$ $16 = y^2$

$y = 0$ $\qquad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ and $(3,0)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ $\qquad$ (1st equation solved for

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ $\qquad$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ $\qquad$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

Find the point(s) of intersection of the graphs for the given equations.

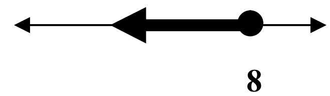
23. $x + y = 8$
    $4x - y = 7$

24. $x^2 + y = 6$
    $x + y = 4$

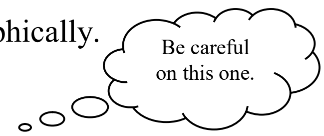
25. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
    $16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

E. Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Solve each equation. Write your answer in interval notation and illustrate your answer graphically.



27. $\frac{x}{2} - \frac{x}{3} > 5$

28. $-4 \le 2x - 3 < 4$

29. $\frac{5}{x+3} \le 2$

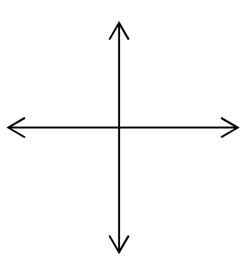
F. Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation. Include a sketch to support your answers. Try to sketch WITHOUT using your TI.

30. $f(x) = x^2 - 5$

D =

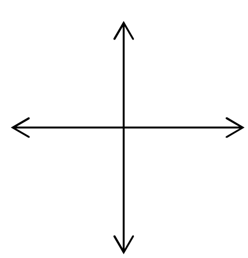
R =



31. $f(x) = -\sqrt{x+3}$

D =

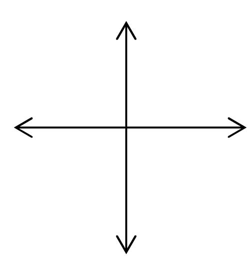
R =



32. $f(x) = 3\sin x$

D =

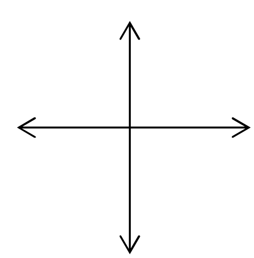
R =



33. $f(x) = \frac{2}{x-1}$

D =

R =

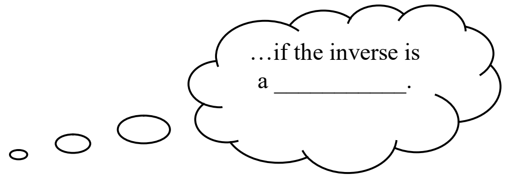


G. Inverses

To find the inverse of a function, simply switch the *x* and the *y* and solve for the new "*y*" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite $f(x)$ as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function. Decide whether the inverse is a function. If it is a function, finish with $f^{-1}(x)$ notation.

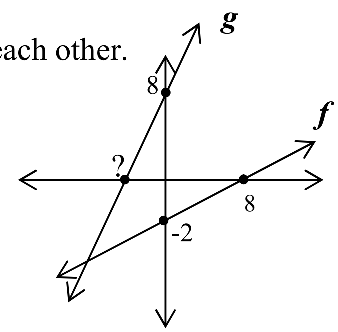
34. $f(x)=\dfrac{2x+1}{x-3}$

35. $f(x)=\dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show the Property of Inverses: $f(g(x))=g(f(x))=x$. *Graphically*, a function and its inverse ____________ through the line _________.

Example:

If: $f(x)=\dfrac{x-8}{4}$ *and* $g(x)=4x+8$ show *f*(*x*) and *g*(*x*) are inverses of each other.



$$f(g(x))=4\left(\frac{x-8}{4}\right)+8$$
$$=x-8+8$$
$$=x$$

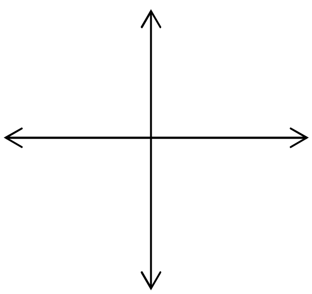
$$g(f(x))=\frac{(4x+8)-8}{4}$$
$$=\frac{4x+8-8}{4}$$
$$=\frac{4x}{4}$$
$$=x$$

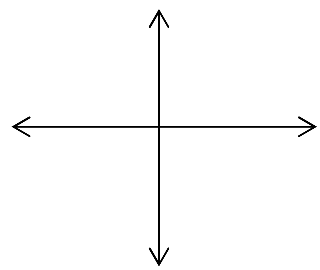
$f(g(x))=g(f(x))=x$ therefore they are inverses of each other.

Use the Property of Inverses to prove *f* and *g* are inverses of each other. Use different colors to sketch *f* and *g* on the given axes and label each function and some corresponding *key* values.

36. $f(x) = \dfrac{x^3}{2} \qquad g(x) = \sqrt[3]{2x}$

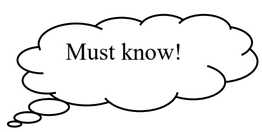
37. $f(x) = 9 - x^2,\ x \geq 0 \qquad g(x) = \sqrt{9 - x}$





H. Equation of a line

Slope intercept form: $y = mx + b$



Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: $x = c$ (slope is *undefined*)

Horizontal line: $y = c$ (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (-2, 5) with a slope of 2/3.

Use ***point-slope form*** to answer the following.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $6y-5x=-6$.

43. Find the equation of a line perpendicular to $y=\frac{5}{6}x-1$ and passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

I. Radian and Degree Measure

Use $\frac{180^\circ}{\pi}$ to get rid of radians and convert to degrees. $\frac{\pi}{12}=\frac{\pi}{12}\cdot\frac{180^\circ}{\pi}$ $=15^\circ$	Use $\frac{\pi}{180^\circ}$ to get rid of degrees and convert to radians. $25^\circ=25^\circ\cdot\frac{\pi}{180^\circ}$ $=\frac{5\pi}{36}$

46. Convert to degrees:

a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

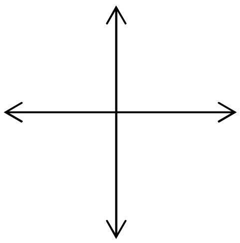
47. Convert to radians:

a. 45° b. -17° c. 237°

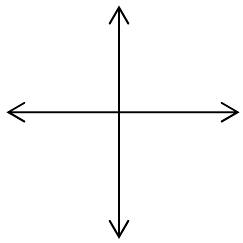
J. Angles in Standard Position

48. Sketch the angle in standard position.

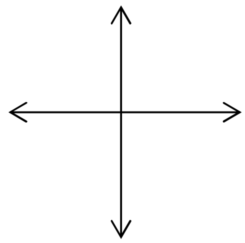
a. $\frac{11\pi}{6}$



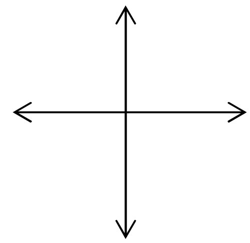
b. $230°$



c. $-\frac{5\pi}{3}$



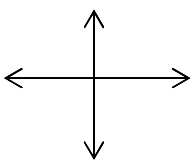
d. 1.8 radians



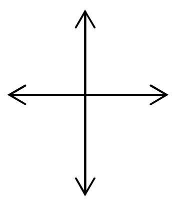
K. Reference Triangles

49. Sketch the angle in standard position. Find the reference angle and label on the diagram.

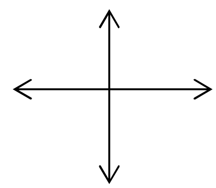
a. $\frac{2}{3}\pi$



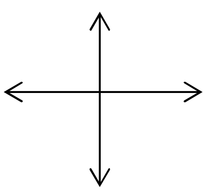
b. $225°$



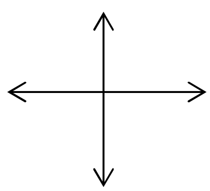
c. $-\frac{3\pi}{4}$



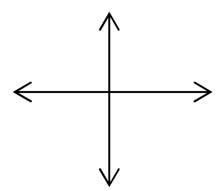
d. $-210°$



e. $\frac{11\pi}{6}$

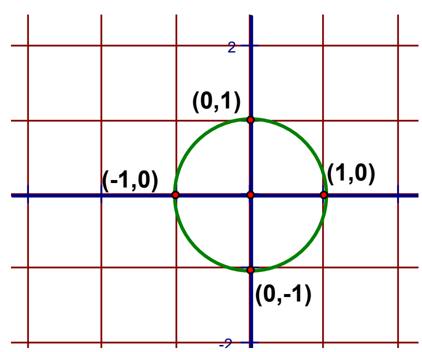


f. $415°$



L. Unit Circle

You can determine the sine or cosine of *quadrantal* angles and other special angles by using the unit circle. The *x*-coordinate of the circle is the cosine and the *y*-coordinate is the sine of the angle. The tangent is the $\frac{y\text{-coordinate}}{x\text{-coordinate}}$.



Example: $\sin 90° = 1$ $\qquad \cos\frac{\pi}{2} = 0$

$\tan\frac{\pi}{4} = \tan 45° = \frac{1}{1} = 1$

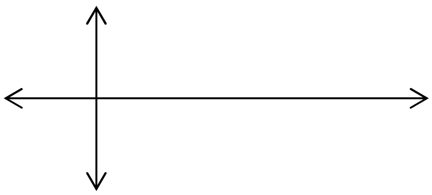
50. *a.*) $\sin 180°$ *b.*) $\cos 270°$ *c.*) $\sin(-90°)$

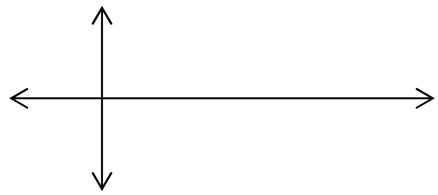
d.) $\sin\frac{3\pi}{4}$ *e*.) $\cos 150^\circ$ *f*.) $\tan(-\pi)$

g.) $\sin\frac{4\pi}{3}$ *h.)* $\cos\frac{7\pi}{4}$ *i.)* $\tan\frac{7\pi}{6}$

M. Graphing Trig Functions

Sketch the graph of $y = \sin x$ and $y = \cos x$ for one full period. Clearly label key values.

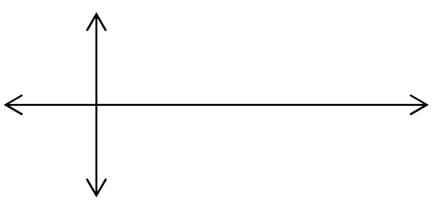




$y = \sin x$ and $y = \cos x$ have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = a\sin(bx + c) + d$, $|a|$ = amplitude, $\frac{2\pi}{b}$ = period, $\frac{c}{b}$ = phase shift (positive $\frac{c}{b}$ shift left, negative $\frac{c}{b}$ shift right) and d = vertical shift.

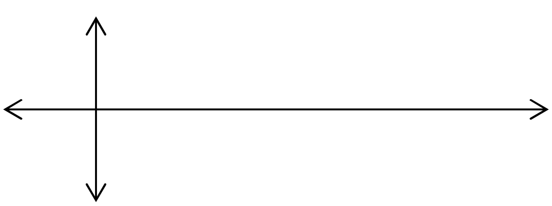
Graph one complete period of the function. Clearly label key values.

51. $f(x) = 5\sin x$

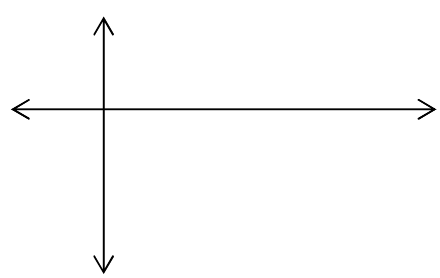


52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$



54. $f(x) = \cos x - 3$



N. Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle if needed, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to use *u-substitution* when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

55. $\sin x = -\dfrac{1}{2}$

56. $2\cos x = \sqrt{3}$

57. $\cos 2x = \dfrac{1}{\sqrt{2}}$

58. $\sin 2x = -\dfrac{\sqrt{3}}{2}$

59. $\sin^2 x = \dfrac{1}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

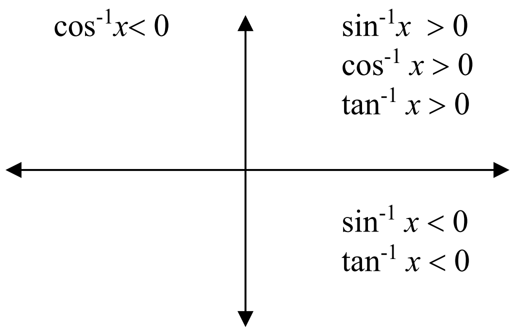
62. $\sin^2 x + \cos 2x - \cos x = 0$

O. Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of two ways:

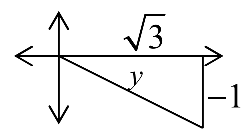
$$f(x) = \arcsin(x) \quad \text{or} \quad f(x) = \sin^{-1}(x)$$

Remember:
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Find $y = \arctan\dfrac{-1}{\sqrt{3}}$ in radians. Basically, find the angle using the domain restrictions such that $\tan y = \frac{-1}{\sqrt{3}}$. A reference triangle might help if you forgot the key values from the unit circle.



This means the reference angle is 30° or $\dfrac{\pi}{6}$. So, $y = -\dfrac{\pi}{6}$ so that it falls in the interval from $\dfrac{-\pi}{2} < y < \dfrac{\pi}{2}$.

Answer: $y = -\dfrac{\pi}{6}$

For each of the following, express the value for "*y*" in radians.

63. $y = \arcsin\left(\dfrac{\sqrt{3}}{2}\right)$

64. $y = \arccos(-1)$

65. $y = \arctan(-1)$

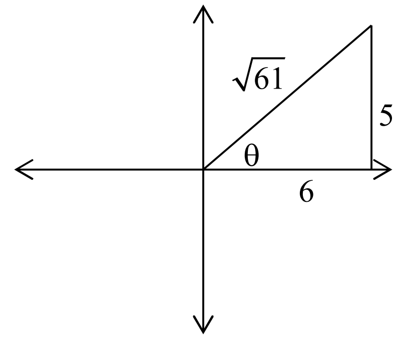
66. $y = \cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$

67. $y = \sin^{-1}\left(\dfrac{-\sqrt{2}}{2}\right)$

68. $y = \cot^{-1}\left(\dfrac{-\sqrt{3}}{3}\right)$

Example: Find the value without a calculator.

$\cos\left(\arctan\frac{5}{6}\right)$ Let $\theta = \left(\arctan\frac{5}{6}\right)$



Draw the reference triangle in the correct quadrant first.

Find the missing side using the Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value *without* using a calculator. Include a diagram that shows the reference triangle in the correct quadrant to support your answer.

69. $\tan\left(\arccos\left(-\frac{2}{3}\right)\right)$

70. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

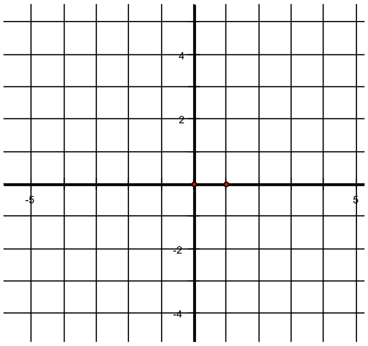
71. $\sin\left(\arctan\left(-\frac{12}{5}\right)\right)$

72. $\cot\left(\sin^{-1}\frac{7}{8}\right)$

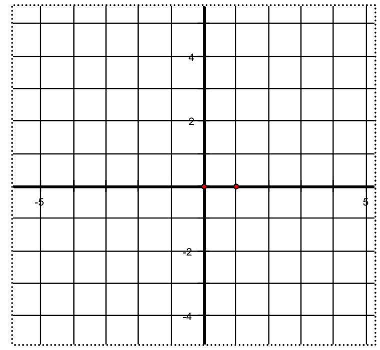
P. Graphing skills.

Find the graphs of the following without relying on your TI. Whenever possible, identify the curve.

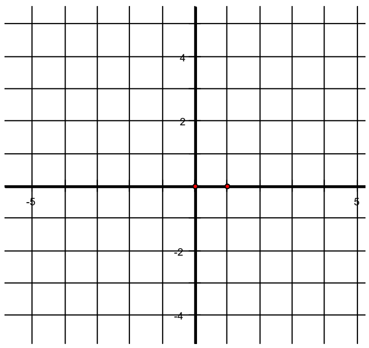
73. $x^2 + y^2 = 16$



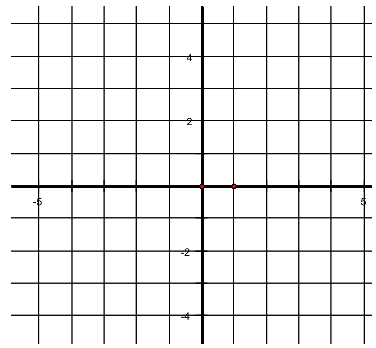
74. $y = \sqrt{16 - x^2}$



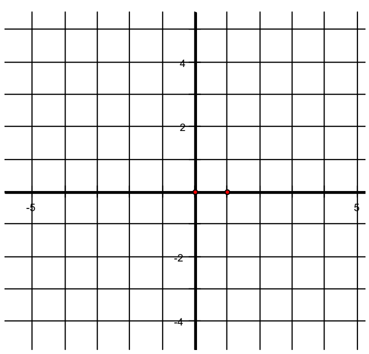
75. $9x^2 + y^2 = 9$



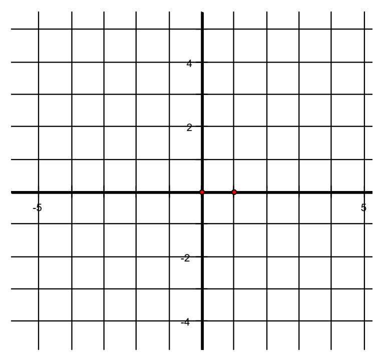
76. $y = -\sqrt{9 - 9x^2}$



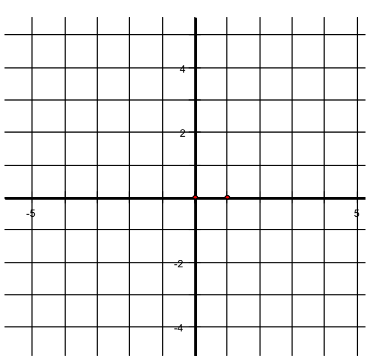
77. $y = x^2 - 6x$



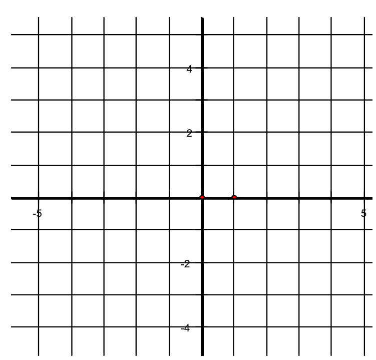
78. $x = y^2 - 6y$



79. $y = \ln x + 2$



80. $y = e^x + 2$



Q. Rational Functions

Vertical Asymptotes and Holes

Set the denominator equal to zero to find the domain restrictions. Factor and Analyze. Find ordered pairs for holes and equations for vertical asymptotes.

81. $f(x) = \dfrac{1}{x^2 - 4}$

82. $f(x) = \dfrac{x+2}{x^2 - 4}$

83. $f(x) = \dfrac{x-1}{x^2 - 3x + 2}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below. This is the “short cut method.”

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the leading coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by synthetic or long division.)

Determine the equations of all Horizontal or Slant Asymptotes. Refer to the above cases.

84. $f(x) = \dfrac{x^2 - 2x + 1}{x^3 + x - 7}$

85. $f(x) = \dfrac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

86. $f(x) = \dfrac{4x^2 - 3x + 2}{x - 7}$

Analyze the end behavior of the rational function by finding each limit as *x* goes to infinity.

RECALL: This is a more analytical process to find Horizontal Asymptotes for a rational function.
Find the highest power of *x* in the denominator.

1. Multiply top and bottom to create “baby fractions.”
2. Analyze your result as you let *x* go to infinity.

87. $\lim\limits_{x\to\infty}\left(\dfrac{2x - 5 + 4x^2}{3 - 5x + x^2}\right) =$

88. $\lim\limits_{x\to\infty}\left(\dfrac{2x - 5}{3 - 5x + x^2}\right) =$

R. Exponents and Logarithms

Recall: Exponential functions and logarithmic functions with the same base are inverses.

- You can solve an exponential equation by taking the log of both sides.
- You can solve a logarithmic equation by exponentiating both sides. [Hint: Before exponentiating both sides…simplify both sides to single terms…no sums or differences.]
- Use $\log_b b^x = x$ and $e^{\ln_e x} = x$

Solve for *x*. Give an exact answer first, then approximate with your TI correct to three decimal places. Show ALL steps.

89. $-6+3e^x = 8$

90. $3\left(5^{x-1}\right)=96$

91. $5^{x+2} = 3^{5-x}$

92. $3^{2x}+3^x-2=0$

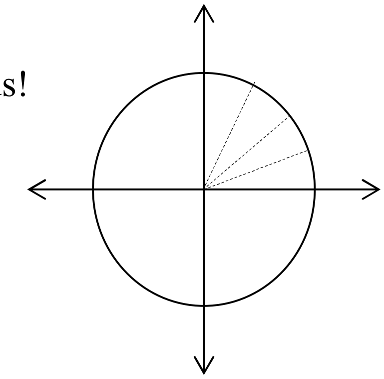
93. $\log_2\left(x-1\right)=5$

94. $\log_3 x+\log_3\left(x-2\right)=1$

95. $\ln\sqrt{x+2}=1$

96. $\ln\left(x-4\right)+\ln\left(x+1\right)=\ln\left(x-8\right)$

The Unit circle and **formulas** you need to know for AP Calculus!



Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Negative Angle Identities:

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Double Angle Identities:

$$\sin 2x = 2\sin x\cos x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Sum/Difference Identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p\log_b m$

Property of equality: If $\log_b m = \log_b n$, then $m = n$

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Slope formula: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Parallel lines: $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$

Perpendicular lines: $l_1 \perp l_2 \Leftrightarrow m_1 = -1/m_2$ or $m_1 \cdot m_2 = -1$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: $\mathrm{A}x + \mathrm{B}y + \mathrm{C} = 0$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$